Automation of a sugar plant diffusion station with a subsystem for resolving conflict modes

Viktor Sidletskyi[1], Oleksandr Kukhar[2], Pavlo Yushchuk[1]

1 - National University of Food Technologies, Kyiv, Ukraine
2 - LLC "TMA Company", Kyiv, Ukraine

Keywords:

Sugar plant
Beet
Game theory
Productivity
Optimization

Article history:

Received 20.08.2023
Received in revised form 14.11.2023
Accepted 11.01.2024

Corresponding author:

Viktor Sidletskyi
E-mail: vmsidletskiy@gmail.com

DOI: 10.24263/2310-1008-2023-11-2-7

Abstract

Introduction. The aim of the present study was to improve of the automated control system of the sugar plant diffusion station, which does not respond adequately to violations of the technological regime when changing the quality indicators of raw beet materials and the rhythm of work.

Materials and methods. The paper considers the data of the diffusion station and the lab for testing of raw materials of the Zhdanivkyi Sugar Plan, Vinnytsia region, Ukraine. To improve the automation system, it is proposed to include a subsystem for resolving conflict modes, and the method of bimatrix non-antagonistic game is used.

Results and discussion. Uncontrolled parameters worsen the quality of raw beet materials and chips, disrupt the movement of chips in the scalding unit and column, and affect machine loading. Contradictions arise: an increase in productivity reduces storage losses, but leads to an increase in sugar losses with the cake because higher throughput reduces the time for optimal extraction. Changes in the condition of the raw beet materials decreases the processing quality and plant productivity.

For the studied sugar plant, changing chip thickness from 6 to 9.5 m/100 grams of beet caused the voltage on the column motors to vary from 135 to 375 V, leading to changes in plant productivity from 2033 to 2358 t/day and sugar losses from 0.39 to 0.65% by weight of processed beets.

An approach to the automation of a diffusion station was proposed. This approach involves supplementing existing systems with a subsystem for resolving conflict modes of operation. The subsystem uses information from the automatic control system, operator inspections, and laboratory data. By applying game theory, it controls the performance of the diffusion station. This algorithm minimizes the total loss of sugar during the storage and processing of beet. It also forms control actions to adjust technological parameters. The methodology involves presenting the problem as a non-antagonistic bimatrix game. In this game, one party aims to increase plant productivity during processing. The other party focuses on reducing losses during processing. The set of strategies involves changing the thickness of the chips. The payoff functions for the parties are productivity and sugar losses during processing.

Conclusion. The use of game theory to optimize the productivity of a sugar plant takes into account sugar losses during beet storage, processing sequence, temperature, beet quality, and changes in sugar losses at different plant productivity.

Introduction

The process of sugar extraction from beet chips is a key part of sugar production. This process significantly impacts the cost of sugar production, energy consumption, and sugar losses (Mabeta et al., 2023; Zayets, 2003). Despite the simplicity of the technological scheme, the dehydration process is complex. It involves different operations including mass transfer, heat transfer, and physicochemical processes.

At most sugar plants, the diffusion station is controlled by an automation system based on modern microprocessor technology (Sidletskyi et al., 2020). These systems reliably maintain the regulated values of the main process parameters (temperature, material balance, level, pressure, and pH). However, existing automation systems cannot always respond adequately to process disruptions (Sadílek et al., 2016; Sidletskyi et al., 2020). This can be explained by the fact that the automation system does not take into account a number of uncontrolled parameters that significantly change the technological regime of the plant and reduces the efficiency of its operation. The quality of beets and chips, the movement of chips in the scalding unit and column, and the loading of the devices are all crucial factors. If there are changes in the technological regime, it is necessary to adjust the technological parameters. Without automatic correction algorithms, the operator must make the decision to change these parameters (Geng et al., 2015; Korobiichuk et al., 2017; Machková et al., 2017; Sidletskyi et al., 2020; Srivastava et al., 2013).

The following issues require additional research:

1. To develop methods for selecting the capacity of the diffusion station and forming a plan for sugar beet processing to minimize total sugar losses during storage and processing.
2. Develop the structure, algorithmic, and software of the subsystem for resolving conflict modes when adjusting the operation of a diffusion station
3. To develop ways to integrate the subsystem for resolving conflict modes into existing automated control systems of the diffusion station.

The aim of the research was to improve the efficiency of the diffusion station for the sugar extraction from beet chips. This was achieved by enhancing the automated control system of the diffusion process through the development and inclusion of a subsystem for resolving conflict modes.

Materials and methods

Object of research

The diffusion station of the Zhdanivkyi Sugar Plan (Vinnytsia region, Ukraine) was used in the study (Fig. 1).

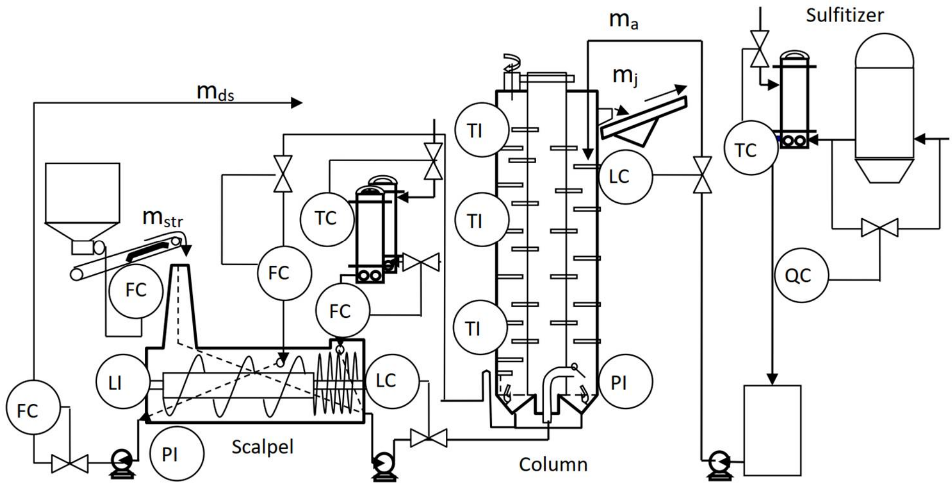

Figure 1. Simplified automation scheme of a diffusion station:
Positions FC - flow control loop, TC - temperature control loop, LC - level control loop, QC - control loop, TI - control loop, RI - control loop, LI - control loop, ma - feed water material flow, m_j - material flow of cake, m_{ds} - material flow of diffusion juice, m_{mstr} - material flow of chips

The automation scheme of the column diffusion unit provides for the implementation of the following control loops (Fig. 1):

- maintaining the ratio of diffusion juice extraction from the mass of processed beets;
- maintaining the ratio of consumption of diffusion and circulation juice;
- maintaining the temperature regime in the scalding unit and the column by stabilizing the temperature of the feed water and circulating juice;
- maintaining the level of juice in the agitator and scalding column.
- To provide the operator with additional information about the state of the technological process, the parameters are monitored:
- juice pH in the column and water treatment station;
- pressure drop on the sieves in the column and scaler;
- specific load on the electric motors of the scalding unit and column drives;
- sugar losses in the pulp;
- quality of the diffusion juice;
- matter content of the diffusion juice at the scalding unit outlet.

Data sources

Sensors and actuators are placed directly on the processing equipment. The quantity and types of these devices are chosen based on the monitoring and control algorithms used at the specific technological site. The automation system in the diffusion department is structured as a hierarchical three-level automated control system. At the first level, there are sensors and actuators; at the second level, there is an industrial controller, and at the third level, there is an automated workstation for the process operator (operator workstation) (Fig. 2).

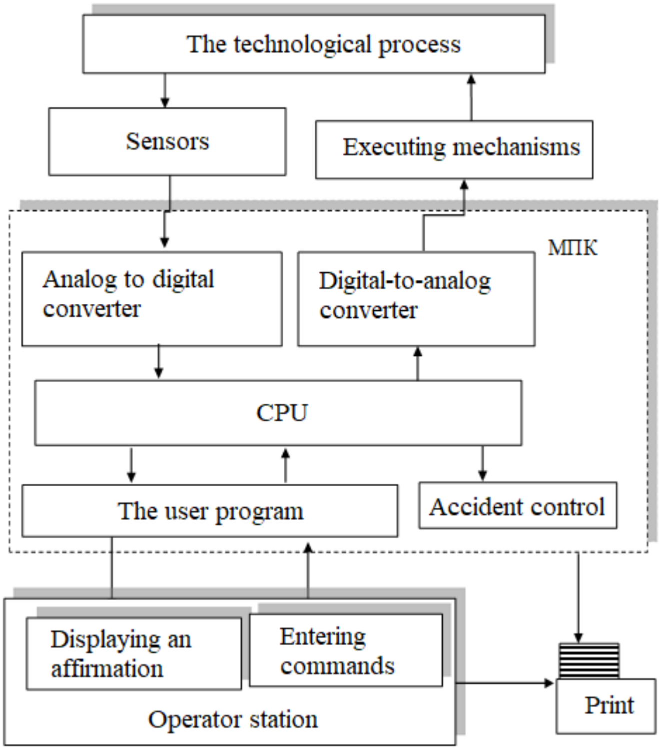

Fig. 2. Functional structure of the automated control system of the diffusion station

Such a system provides the following functions: polling and subsequent processing of signals from sensors, execution of operating control algorithms, production and emergency alarms, development of control actions on actuators, transmission of signals from sensors, positions of actuators, diagnostic signals to the operator station, providing the operator with the ability to change the set values for regulators and manual control of actuators, archiving of values of technological parameters and parameters of equipment operation, and maintaining.

Research methods

The study is a bimatrix non-antagonistic repeated game for two players with a Nash equilibrium search (Cruz et al., 2005; Du, 2008; Kontogiannis et al., 2009).

Results and discussion

Conflicting regimes of operation of the diffusion station.

A complex indicator of disturbances in the operation of a diffusion plant is the decrease of its quality indicators, which are determined in the process of laboratory analysis of diffusion juice, pulp or determination of total sugar losses (Zayets, 2003). The complexity of the operator's work lies in the fact that deviations in the technological regime and deterioration of the process quality indicators can be caused by various reasons (Korobiichuk et al., 2018; Srivastava et al., 2013). At the same time, decision-making to eliminate them is also ambiguous.

For example, as noted above, the quality of diffusion juice can be changed by the pumping coefficient α, which in turn can be changed either by the amount of diffusion juice mdc that is pumped out or the amount of beet chips mc that is fed for processing, i.e. by changing the productivity of the apparatus (Sidletskyi et al., 2020).

At first glance, changing the productivity can be done very simply by changing the speed of the column's conveying body. But the column's performance also depends on the specific load of the column and the conditions of chip movement in it:

$$Y = f_y\left(R, V, Z\right), \tag{2}$$

where *Y* is the productivity of the diffusion apparatus, *R* is the specific load, *V* is the shaft speed, and *Z* is the chip movement in the apparatus.

In turn, the specific load *R* depends on: shaft speed *V*, chip flow *F*, level in the machine *L*, chip quality *W*, and is characterized by the load of the electric motor I) (Sidletskyi et al., 2020):

$$R = f_r\left(V, F, L, W, I\right) \tag{3}$$

In this case, the movement of chips in the apparatus *Z* depends on: the specific load in the apparatus *R*, the pH of the water *P*, the quality of the chips *W*, and the temperature *T* (Sidletskyi et al., 2020):

$$Z = f_z\left(R, P, W, T\right) \tag{4}$$

To maximize the column's performance, it is necessary to operate at the maximum turbine speed, maintain the maximum specific load and chip movement in the apparatus. Considering 2, 3 and 4:

$$\begin{cases} R = f_r\left(V, F, L, W, I, O_1\right) \\ Z = f_z\left(R, P, W, T, O_2\right) \\ Y = f_y\left(R, V, Z\right) \end{cases} \tag{5}$$

A graph was developed (Fig. 3) showing the effect of specific load and chip movement on the productivity of the diffusion station.

The graph shows that it is impossible to change the productivity by the speed of the tube without taking into account the state of the chips, which affects the load and movement of the chips in the apparatus.

Therefore, the role of the operator-technologist in the diffusion station control system is very important. The quality of management depends on his/her professional qualities and the time allotted for decision-making. An untimely or erroneous decision can lead to damage to process equipment, an increase in the cost of production, and a deterioration in its quality. This is especially true for sugar production, which is seasonal in nature, in which the operator's experience either does not have time to accumulate or during the period when the sugar plant is not working, his professional skills are lost.

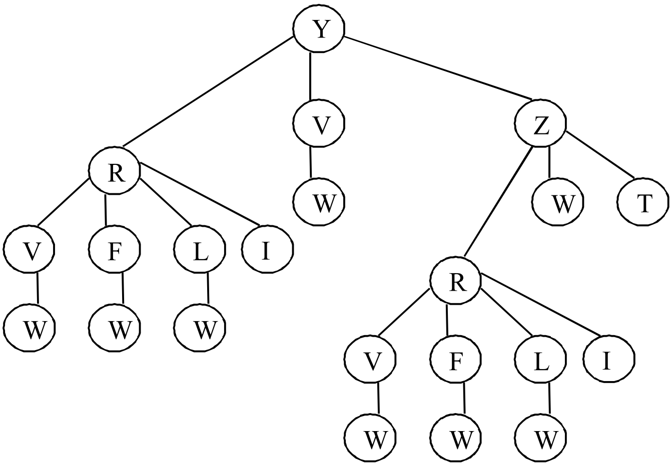

Fig. 3. Influence of factors on the performance of the diffusion apparatus

The analysis shows that the performance of the diffusion station significantly affects both the quality of the diffusion process during sugar beet processing and sugar losses during the storage of raw materials. At the same time, there are some contradictions in decision-making. If there is a need to reduce sugar losses during storage, it is necessary to increase the rate of processing of raw materials, i.e. to increase the productivity of the diffusion department. But this can lead to an increase in sugar losses in the pulp and a deterioration in the quality of the diffusion process. On the other hand, changes in raw material condition indicators lead to changes in the technological conditions of the extraction process and significantly affect the quality indicators of the diffusion process. This may require a change in the residence time of the chips in the column, which will change the productivity of the plant and disrupt its rhythmicity (Geng et al., 2015; Sadílek et al., 2016; Sidletskyi et al., 2020). And this, in turn, will affect the shelf life of raw materials and, accordingly, sugar losses during storage. Therefore, the choice of performance control algorithms is an important task.

It's important to note that simply reducing individual sugar losses during storage and processing may not always achieve the desired outcome. For example, if bad weather causes higher sugar losses in stored beets, increasing the processing rate may seem like a solution, but it can actually lead to higher sugar losses during processing. In some cases, it might be better to adjust the processing methods to minimize overall sugar losses, even if it means accepting higher losses in certain areas, like the pulp (Sidletskyi et al., 2020).

There are two ways to change the performance of a diffusion station (Sidletskyi et al., 2020):

- increasing the voltage on the motors of the tube column, which leads to an increase in the speed of chips passing through the machine and reduces the time spent in the machine;
- reduction of chip length (increase in geometric dimensions), which leads to the need to increase the time spent in the machine. This is because coarser chips move better in the machine, and therefore the machine can be filled with more chips. However, this increases the thickness of the diffusion layer, which requires an increase in the diffusion time, i.e. a decrease in the voltage on the column motors.

– Therefore in the process of finding management decisions to change the productivity of raw material processing while maintaining sugar losses within specified limits, a non-antagonistic conflict situation arises, in which two parties can be distinguished (Abalo et al., 2004; Enkhbat et al., 2021; Fletcher et al., 2007; Kontogiannis et al., 2009):
– The first is interested in increasing productivity by increasing the voltage on the column motors;
– The second is interested in reducing losses during sugar beet processing by increasing the length of the chips.

The length of the chips depends on the quality of the raw material and the setting of the beet cutters (Sidletskyi et al., 2020; Zayets, 2003), i.e. it is a parameter that cannot be controlled in an operational manner. Therefore, the task of the system will be to find such values of motor voltage at which the maximum productivity of the station will be achieved for the available chip length, provided that sugar losses during storage are not affected.

Presentation of station operation modes in the form of a non-antagonistic game

Assuming that each value of voltage on the column motors v and chip length d will correspond to both productivity A and sugar losses B, it is necessary to find a situation in which productivity is maximized and losses are minimized:

$$\begin{cases} A = f_1(v,d) \\ B = f_2(v,d) \end{cases} \quad A \to \max,\ B \to \min \tag{6}$$

No mathematical expression was found that could be used to obtain the dependence of productivity on the voltage on the column motors and chip length, since a change in the voltage on the column motors does not lead to a proportional change in productivity (Geng et al., 2015; Sadílek et al., 2016; Sidletskyi et al., 2020; Zayets, 2003). This can be explained by the fact that productivity primarily depends on the processes that affect the movement of chips, which, as shown in the previous sections, depend on a significant number of factors. It should be noted that in the process of operation, the operator always finds a mode that can be called steady-state, in which the specified performance indicators of the diffusion station were achieved at different voltage on the shaft of the column motors, different chip length and at different productivity (Sidletskyi et al., 2020). Based on this, using the experimental data of the diffusion station operation, a matrix was formed (Table 1), which could be used to determine what productivity was characteristic of the voltage of the column motors v and the chip length *l* (Sidletskyi et al., 2020).

Table 1

Productivity values (*a*) depending on column motor voltage and chip length

Voltage of column motors	Chip length l				
	l_1	…	l_i	…	l_n
	Diffusion station capacity, tons				
v_1	a_{11}	…	a_{1j}	…	a_{1n}
…	…	…	…	…	…
v_j	a_{1j}	…	a_{ij}	…	a_{in}
…	…	…	…	…	…
v_m	a_{1m}	…	a_{mi}	…	a_{mn}

The symbols (...) are used to indicate that a certain part of the table has been omitted, but similar information follows.

Similarly, for sugar losses during processing, a matrix was formed (Table 2) that characterized sugar losses β depending on the voltage of the column motors v and chip length l.

Table 2

Sugar loss values (β) depending on column motor voltage and chip length

Voltage of column motors	Chip length l				
	l_1	...	l_i	...	l_n
	Sugar loss				
v_1	β_{11}	...	β_{1j}	...	β_{1n}
...	...	...	...	...	...
v_j	β_{1j}	...	β_{ij}	...	β_{in}
...	...	...	...	...	...
v_m	β_{1m}	...	β_{mi}	...	β_{mn}

To solve this problem, we can consider condition (6) as a non-antagonistic bimatrix game:

$$G = \left(N, \{X_i\}_{i\in N}, \{H_i\}_{i\in N}\right) \tag{7}$$

in which $N=\{1,2,...n\}$ is the set of players, X_i is the set of strategies of player i, H_i is the payoff function of player i.

We view the recycling process as a game of G, in which there are two sides.

$$G = \left(X_{11}, X_{12}, H_{11}, H_{12}\right) \tag{8}$$

One party is interested in increasing the productivity of processing and for which the set of strategies is to change the voltage on the column motors $X_1=\{v_1,v_2,...,v_m\}$, the other party is interested in reducing the losses during processing and for which the set of strategies is to change the length of the chips $X_2=\{d_1,d_2,...,d_n\}$. The gain functions for the parties will be $H_1=P(v,d)$ i $H_2=C_2(v,d)$ for productivity and sugar losses during processing, respectively. Then the tables of sugar losses during processing and processing productivity, depending on the voltage and chip length, will take the form of game matrices G:

$$H_1 = \begin{array}{c|cccccc|} & l_1 & \dots & l_i & \dots & l_n \\ \hline v_1 & a_{11} & \dots & a_{i1} & \dots & a_{n1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ v_j & a_{1j} & \dots & a_{ij} & \dots & a_{nj} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ v_m & a_{1m} & \dots & a_{im} & \dots & a_{nm} \end{array}$$

$$H_2 = \begin{array}{c|cccccc|} & l_1 & \dots & l_i & \dots & l_n \\ \hline v_1 & \beta_{11} & \dots & \beta_{i1} & \dots & \beta_{n1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ v_j & \beta_{1j} & \dots & \beta_{ij} & \dots & \beta_{nj} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ v_m & \beta_{1m} & \dots & \beta_{im} & \dots & \beta_{nm} \end{array} \tag{9}$$

For a game *G*, the equilibrium situation is the value of *v* and *d* for which the condition is satisfied:

$$G=\left(v,d\right)\rightarrow\left\{\left(v^{*},d^{*}\right)\left|\begin{matrix}H_{1}\left(v,d^{*}\right)=\max\limits_{X_{11}}H_{1}\left(v,d\right)^{*}\\H_{2}\left(v^{*},d\right)=\min\limits_{X_{12}}H_{2}\left(v^{*},d\right)\end{matrix}\right.\right\}, \quad (10)$$

Assuming that for a game *G* the full set of probabilities of the first party using its pure strategies is mixed strategy $X_{11}=\{v_1,v_2,...,v_m\}$ and $X_{12}=\{d_1,d_2,...,d_n\}$ - the mixed strategies of the second party. Then the average payoffs of the first and second parties will be equal:

$$\begin{matrix}G\left(H_{1},v,d\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}v_{i}d_{j}\\G\left(H_{2},v,d\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}\beta_{ij}v_{i}d_{j}\end{matrix}. \quad (11)$$

The equilibrium situation for a given bimatrix game will be a pair (*v, d*) of mixed strategies of the first and second party that satisfy the inequalities:

$$\begin{matrix}\sum_{j=1}^{n}a_{ij}d_{j}\le\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}v_{i}d_{j}\quad\left(i=1,\ldots,m\right)\\\sum_{i=1}^{m}\beta_{ij}v_{i}\le\sum_{i=1}^{m}\sum_{j=1}^{n}\beta_{ij}v_{i}d_{j}\quad\left(j=1,\ldots,n\right)\end{matrix}. \quad (12)$$

According to [114], for a situation (v^0, d^0) to be an equilibrium situation in the mixed strategies of a bimatrix game G, it is necessary and sufficient that there exist sets $H_1^0\subseteq H_1$, $H_2^0\subseteq H_2$ and the numbers u_1, u_2, for which the conditions are met:

$$\begin{cases}\sum\limits_{j\in H_{22}^{0}}a_{ij}v_{j}^{0}=u_{1} & \forall i\in H_{1}^{0},\\\sum\limits_{j\in H_{22}^{0}}a_{ij}v_{j}^{0}\le u_{1} & \forall i\notin H_{1}^{0},\\\sum\limits_{j\in H_{22}^{0}}v_{j}^{0}=1,v_{j}^{0}\ge1 & \forall j\in H_{2}^{0},\end{cases} \quad (13)$$

$$\begin{cases}\sum\limits_{j\in H_{22}^{0}}d_{j}^{0}\beta_{ij}=u_{2} & \forall j\in H_{2}^{0},\\\sum\limits_{j\in H_{22}^{0}}d_{j}^{0}\beta_{ij}\le u_{2} & \forall j\notin H_{2}^{0},\\\sum\limits_{j\in H_{22}^{0}}d_{j}^{0}=1,d_{j}^{0}\ge1 & \forall i\in H_{1}^{0},\end{cases}. \quad (14)$$

Resolving conflicting modes of operation on the example of *Zhdanivkyi Sugar Plant*

To verify the approach, we took archival data from the Zhdanivkyi Sugar Plant's control system and information from the lab for testing of raw materials. Based on these data, matrices of the dependence of productivity and sugar losses during beet processing on the voltage of the column motors (the time the chips stay in the apparatus) and the length of the chips were constructed.

The payoff matrices for each side are given in the form of tables; for the first side, the net strategies are $X_1=\{135, 145, 155, 165, 175, 185, 195, 205, 215, 225, 235, 245, 255, 265, 275, 285, 295, 305, 315, 325, 335, 345, 355, 365, 375\}$, and for the other side, pure strategies will be $X_2=\{6, 6.5, 7, 7.5, 8, 8.5, 9, 9.5\}$.

Table 3

Productivity depending on column motor voltage and chip length

		Chip length, m/100 g							
		6.0	**6.5**	**7.0**	**7.5**	**8.0**	**8.5**	**9.0**	**9.5**
		Diffusion station capacity, tons/day							
Voltage of column diffusion apparatus motors, V	135	2033	2109	2150	2008	2075	2096	2082	2023
	145	2020	2021	2152	2176.98	2027	2113	2055	2088
	155	2213	2089	2143	2020	2049	2063	2091	2091
	165	2142	2201	2168	2093	2025	2025	2085	2139
	175	2058	2233	2137	2046	2157	2019	2030	2021
	185	2198	2011	2171	2022	2010	2063	2164	2117
	195	2103	2059	2050	2185	2186	2078	2201	2111
	205	2005	2165	2146	2236	2141	2109	2184	2121
	215	2226	2269	2120	2103	2072	2003	2082	2146
	225	2169	2048	2274	2278	2216	2118	2104	2127
	235	2303	2322	2149	2171	2206	2224	2032	2019
	245	2282	2205	2074	2029	2089	2041	2232	2103
	255	2267	2316	2235	2082	2218	2162	2196	2103
	265	2122	2003	2256	2075	2158	2184	2156	2176
	275	2287	2032	2007	2065	2115	2169	2197	2045
	285	2036	2214	2325	2261	2008	2133	2284	2102
	295	2038	2413	2327	2160	2294	2207	2160	2066
	305	2471	2261	2212	2116	2170	2331	2164	2151
	315	2153	2047	2058	2343	2313	2283	2229	2214
	325	2256	2038	2002	2326	2142	2020	2321	2070
	335	2330	2451	2046	2332	2341	2030	2164	2049
	345	2448	2305	2429	2064	2144	2081	2185	2260
	355	2444	2080	2294	2039	2304	2112	2103	2341
	365	2479	2408	2348	2321	2181	2061	2049	2231
	375	2278	2519	2158	2333	2138	2055	2027	2358

Table 4

Sugar loss depending on column motor voltage and chip length

		Chip length, m/100 g							
		6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5
		Sugar losses during beet processing, % by weight of beet							
The voltage of column diffusion apparatus motors, V	135	0.39	0.35	0.28	0.26	0.21	0.27	0.23	0.21
	145	0.47	0.34	0.35	0.25	0.23	0.24	0.23	0.23
	155	0.43	0.42	0.30	0.29	0.28	0.28	0.24	0.23
	165	0.42	0.34	0.40	0.28	0.32	0.26	0.30	0.22
	175	0.56	0.45	0.36	0.38	0.33	0.26	0.29	0.26
	185	0.60	0.36	0.34	0.33	0.31	0.37	0.29	0.30
	195	0.53	0.51	0.38	0.32	0.35	0.31	0.30	0.36
	205	0.50	0.51	0.47	0.37	0.33	0.31	0.36	0.33
	215	0.61	0.52	0.51	0.45	0.39	0.43	0.33	0.37
	225	0.74	0.57	0.45	0.38	0.36	0.33	0.34	0.36
	235	0.55	0.52	0.43	0.40	0.36	0.43	0.43	0.32
	245	0.73	0.69	0.53	0.56	0.37	0.43	0.46	0.39
	255	0.59	0.55	0.58	0.43	0.52	0.45	0.36	0.48
	265	0.60	0.72	0.62	0.53	0.42	0.42	0.38	0.47
	275	0.91	0.56	0.64	0.47	0.53	0.48	0.48	0.36
	285	0.83	0.68	0.60	0.62	0.53	0.54	0.52	0.38
	295	0.69	0.71	0.62	0.55	0.48	0.44	0.46	0.49
	305	0.81	0.85	0.58	0.53	0.46	0.48	0.45	0.42
	315	0.96	0.81	0.72	0.52	0.60	0.51	0.49	0.49
	325	0.84	0.84	0.73	0.58	0.50	0.67	0.52	0.59
	335	1.00	0.81	0.64	0.67	0.65	0.56	0.56	0.57
	345	1.08	0.89	0.68	0.78	0.70	0.52	0.56	0.63
	355	1.03	0.86	0.85	0.71	0.68	0.57	0.66	0.55
	365	1.19	1.02	0.64	0.69	0.57	0.57	0.50	0.60
	375	0.88	1.04	0.75	0.81	0.68	0.59	0.68	0.65

To find an equilibrium situation in biquadratic games, algorithms for rejecting strictly dominant strategies are used (Abalo et al., 2004; Enkhbat et al., 2021; Fletcher et al., 2007; Kontogiannis et al., 2009). To do this, it is necessary to find the dominant rows of the H_1 matrix and columns in the H_2, matrix, while distinguishing between strict and weak dominance.

In a bimatrix game, the strategy of the first side i_1 is strictly dominant over strategy i_2 ($i_1 > i_2$) on the set $\overline{H_2} \subseteq H_2$, if $a_{i_1 j} > a_{i_2 j}\ \forall j \in \overline{H_2}$. Weak dominance ($i_1 \geq i_2$), if $a_{i_1 j} \geq a_{i_2 j}\ \forall j \in \overline{H_2}$. Similarly, for the second party, strategy j_1 strictly dominates strategy j_2 ($j_1 > j_2$) on the set $\overline{H_1} \subseteq H_1$, if $d_{ij_1} > d_{ij2}\ \forall i \in \overline{H_1}$ and weak dominance ($j_1 \geq j_2$) if $d_{ij_1} \geq d_{ij2}\ \forall i \in \overline{H_1}$ $d_{ij_1} \geq d_{ij2}\ \forall i \in \overline{H_1}$ (Abalo et al., 2004; Enkhbat et al., 2021).

Using the definition of dominant strategies, the procedure of sequential exclusion of strictly dominant strategies was carried out. In the process of excluding dominant strategies, two sequentially nested sets were constructed $H_1 = H_1^1 \supseteq H_1^2 \supseteq \cdots \supseteq H_1^k$ and $H_2 = H_2^1 \supseteq H_2^2 \supseteq \cdots \supseteq H_2^k$.

If the following conditions are met for *l=1,…, к-1*:

$$\forall i_2 \in H_1^i \setminus H_1^{i+1} \exists i_1 \in H_1^{i+1} : i_1 > i_2 \text{ на } H_2^1; \tag{15}$$

$$\forall j_2 \in H_2^i \setminus H_2^{i+1} \exists j_1 \in H_2^{i+1} : j_1 > j_2 \text{ на } H_1^1. \tag{16}$$

After the procedure of eliminating all dominant strategies, mixed Nash equilibria remain (Du, 2008; Cruz et al., 2005):

1. If the set $\overline{Z} = \overline{H_1} \times \overline{H_2}$ strictly dominates the set $Z = H_1 \times H_2$ in mixed strategies, then for any equilibrium situation (v^0, d^0) conditions (Abalo et al., 2004; Cruz et al., 2005; Du, 2008; Enkhbat et al., 2021; Kontogiannis et al., 2009) are met:

$$i \notin \overline{H_1} \Rightarrow v_i^0 = 0 \tag{17}$$

$$j \notin \overline{H_2} \Rightarrow d_j^0 = 0 \tag{18}$$

2. If the set $\overline{Z} = \overline{H_1} \times \overline{H_2}$ weakly dominates the set $Z = H_1 \times H_2$ in mixed strategies and ($\overline{v}$, $\overline{d}$) is an equilibrium situation for a game with matrices (Du, 2008; Cruz et al., 2005; Enkhbat et al., 2021; Kontogiannis et al., 2009):

$$\overline{A} = \left(a_{ij}\right)_{i \in \overline{H_1}\ j \in \overline{H_2}}, \quad \overline{B} = \left(\beta_{ij}\right)_{i \in \overline{H_1}\ j \in \overline{H_2}} \tag{19}$$

Then there will be mixed strategies:

$$v^0 : v_i^0 = \begin{cases} \overline{v}_i, & i \in \overline{H_1}, \\ 0, & i \in H_1 \setminus \overline{H_1}; \end{cases} \qquad d^0 : d_j^0 = \begin{cases} \overline{d}_j, & j \in \overline{H_1}, \\ 0, & j \in H_1 \setminus \overline{H_1}, \end{cases} \tag{20}$$

where v^0, d^0 are equilibrium situation for the game G_1.

For the technological process, it is best to have the longest residence time and the longest chip length. However, the increase in chip length increases the load on the machine (Cruz et al., 2005; Du, 2008; Enkhbat et al., 2021; Kontogiannis et al., 2009), and therefore it is necessary to reduce the amount of chips in the machine, i.e. either to reduce the residence time or to feed coarser chips. In turn, the supply of coarser chips (reduction in length) or reduction of the chip residence time in the apparatus leads to an increase in sugar consumption with pulp (Sidletskyi et al., 2020).

Block diagram of the diffusion station control with the module for controlling the performance of the diffusion station

The analysis of the functions that should be performed by the module for generating recommendations for managing processing productivity made it possible to identify the following elements that should comprise its structure:

- a block for assessing the condition of raw materials, which calculates possible sugar losses during storage of raw materials;
- a block for calculating the plan for supplying raw materials for processing, taking into account the quality indicators of raw materials and sugar losses during storage;

– a block for calculating processing capacity, taking into account raw material losses during storage.

The solution to this problem can be achieved by including an additional module for controlling the performance of the diffusion station in the automation system of Fig. 1 (Sidletskyi et al., 2020).

Data from the diffusion station is used to analyze productivity and sugar losses during processing, considering column motor voltage and chip length. This analysis aims to find optimal motor voltage values for maximizing productivity and minimizing sugar losses.

Meanwhile, the raw material lab provides information on raw material quantity and condition for storage. This data is input into units calculating sugar losses during storage and planning raw material supply.

For each level of productivity, we calculate storage and processing losses to find the productivity level with the lowest total losses. Then, based on the productivity and processing losses, we determine the voltage needed for the column motors. The analysis of the diffusion station's efficiency highlights productivity as the main factor influencing total sugar losses. Using formulas for average daily weight and sugar losses of sugar beets, along with stock management methodology, we derived a formula to predict sugar beet state changes based on daily temperature. The analysis also shows that productivity can be adjusted by changing column motor voltage or chip length. The productivity control system aims to find optimal motor voltage values for maximum plant productivity, considering chip length and minimizing sugar losses during storage, using game theory. It's worth noting that other researchers have tackled similar problems using game theory (Abalo et al., 2004; Enkhbat et al., 2021; Fletcher et al., 2007; Kontogiannis et al., 2009).

Conclusions

The use of game theory in calculating the optimal productivity of a sugar factory allows for accounting for sugar losses during storage and their dependency on: the sequence of processing each batch, ambient temperature, raw material quality indicators. It also enables considering changes in sugar losses ranging from 0.39% to 0.65% of the processed beet mass during processing when productivity varies between 2033 and 2358 tons per day, by adjusting the geometric parameters of the chips within the range of 6 to 9.5 meters per 100 grams of chips and the residence time of the chips in the apparatus, which depends on the voltage on the motors of the column diffusion apparatus ranging from 135 to 375 V.

References

Abalo K., Kostreva M. (2004) Some existence theorems of Nash and Berge equilibria, *Applied Mathematics Letters*, 17, pp. 569–573

Cruz J., Tan X. (2005) Price strategies in dynamic duopolistic markets with deregulated electricity supplies using mixed strategies, *Decision Support Systems*, 40(3–4), pp. 439–447.

Du Y. (2008), On the complexity of deciding bimatrix games similarity, *Theoretical Computer Science*, 407(1-3), pp. 583–586.

Enkhbat R., Sukhee B. (2021), Optimization approach to Berge equilibrium for bimatrix game, *Optimization Letters*, 15, pp. 711–718, https://doi.org/10.1007/s11590-020-01688-8

Fletcher P., Joshi S. (2007), Riemannian geometry for the statistical analysis of diffusion tensor data, *Signal Processing*, 87(2), pp. 250–262, https://doi.org/10.1016/j.sigpro.2005.12.018

Geng G., Yang J. (2015) Sugar beet production and industry in China, *Sugar Technology*, 17(1), pp. 13–21.

Ivanov V., Shevchenko O., Marynin A., Stabnikov V., Gubenia O., Stabnikova O., Shevchenko A., Gavva O., Saliuk A. (2021), Trends and expected benefits of the breaking edge food technologies in 2021–2030, *Ukrainian Food Journal*, 10(1), pp. 7–36, https://doi.org/10.24263/2304-974X-2021-10-1-3

Kontogiannis S., Panagopoulou P., Spirakis P. (2009), Polynomial algorithms for approximating Nash equilibria of bimatrix games, *Theoretical Computer Science*, 410(17), pp 1599–1606, https://doi.org/10.1016/j.tcs.2008.12.033

Korobiichuk I., Ladanyuk A., Shumyhai D., Boyko R., Reshetiuk V., Kamiński M. (2017), How to increase efficiency of automatic control of complex plants by development and implementation of coordination control system, *Advances in Intelligent Systems and Computing*, 543, pp. 189–195, https://doi.org/10.1007/978-3-319-48923-0_23

Korobiichuk I., Ladanyuk A., Zaiets N., Vlasenko L., (2018), Modern development technologies and investigation of food production technological complex automated systems, *International Conference on Mathematics and Science Education (ICMScE 2018)*, pp. 52–57, https://doi.org/10.1145/3185066.3185075

Korobiichuk I., Lutskaya N., Ladanyuk A., Naku S., Kachniarz M., Nowicki M., Szewczyk R. (2017), Synthesis of optimal robust regulator for food processing facilities, *Advances in Intelligent Systems and Computing*, 550, pp. 58–66, https://doi.org/10.1007/978-3-319-54042-9_5

Machková H., Sato A. (2017), Analysis of competitiveness of Belgian sugar industry, *Cukrovarnicke a Reparske*, 133(12), pp. 390–392.

Mabeta J., Smutka L. (2023), Historical and contemporary perspectives of the sugar industry in sub-Saharan Africa, *Ukrainian Food Journal*, 12(1), pp. 157–171, https://doi.org/10.24263/2304-974X-2023-12-1-12

Sadílek T., Zadrazilová D. (2016), Current trends in German sugar industry, *Listy Cukrovarnicke a Reparske*, 132(12), pp. 390–393.

Sidletskyi V., Korobiichuk I., Ladaniuk A., Elperin I., Rzeplińska-Rykała K. (2020), Development of the structure of an automated control system using tensor, *Techniques for a Diffusion Station, Automation 2019. Advances in Intelligent Systems and Computing*, 920, pp. 175–185, https://doi.org/10.1007/978-3-030-13273-6_18

Srivastava D., Mohanty B., Bhargava R. (2013), Modeling and simulation of MEE system used in the sugar industry, *Chemical Engineering Communications*, 200, pp. 1089–1101, https://doi.org/10.1080/00986445.2012.737876

Zayets Y. (2003), The Ukrainian sugar industry - current situation and development trends, *Sugar Industry-Zuckerindustrie*, 128 (8), pp. 571–580.