Modeling of heat transfer processes by periodic perturbation of freely flowing liquid film flowing along vertical surface with large-amplitude waves

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Abstract

Introduction. The aim of the study was the theoretical and experimental determination of the influence of periodic disturbances of the liquid film surface on the intensity of heat transfer during evaporation.

Materials and methods. The theoretical results were obtained based on mathematical modeling of heat transfer processes in flowing liquid films with disturbance of its surface by large waves. Physical modeling was performed on an experimental stand, on which heat exchange processes in the film were reproduced in a 9-m-long pipe with a diameter of 30 mm, which was heated by dry saturated steam. The heat flow was measured at the discrete-local level.

Results and discussion. The model of heat exchange in a liquid film flowing down a vertical surface is presented, which is cyclically mixed either due to the passage of low-frequency waves of large amplitude, or due to a system of mechanical obstacles in the way of the film movement. The analytical expression for the temperature field in the turbulent film is obtained by the approximate solution of the convective heat conduction equation with the boundary conditions under which vapor formation occurs on the free surface of the film. An analytical expression for the heat transfer coefficient is presented, which includes, as parameters, the distance between the heat transfer intensifiers and the maximum value of the turbulence function, which is proportional to the height of the mechanical turbulators. For film currents flowing in long, smooth heat exchange channels, the role of the parameters is played by the distance between the crests of large waves and their amplitude. The practical implementation of the obtained results of the mathematical modeling of heat transfer in the processes of cyclic disturbance and restoration of the hydrodynamic structure of flowing films is demonstrated by generalization on its basis of the data of experimental studies of heat exchange in a flowing film of water in a pipe with a diameter of 30 mm and a height of 9 m in the range of changes in the mass density of irrigation 0.16–0.72 kg/m sec with.

Conclusions. A mathematical model of heat exchange in a liquid film flowing down a vertical surface disturbed either by low-frequency waves of large amplitude or by a system of ring intensifiers of heat exchange has been developed.

Introduction

Thanks to the action of gravity, the movement of liquid flowing down the vertical surface of the film occurs in the absence of stagnant zones, and heat and mass exchange processes, due to the small thickness of the films, proceed rapidly in small volumes and are characterized by high intensity. These factors become especially important during the concentration of viscous thermolabile food solutions under rarefaction, where the hydrostatic temperature depression, which is absent in film evaporators, increases rapidly. At the same time, during the movement of highly concentrated thick films of solutions, the intensification of heat exchange processes becomes relevant.

Intensification of heat exchange can be implemented with the help of various types of mechanical turbulating inserts, or the implementation of forced low-frequency pulsations of the fluid flow. When using long evaporation channels, low-frequency deformed waves of high amplitude develop, which make the main contribution to the process of heat exchange intensification in films. The processes of heat transfer intensification by large waves are given much attention by researchers (Choudhury et al., 2023; Gourdon et al., 2015; Kostoglou et al., 2010; Malamatairis et al., 2008; Mascarenhas et al., 2013; Ruyer-Quil et al., 2005).

The amplitude of large waves is 2–3 times greater than the average thickness of the film. In addition, the speed of large waves is 1.5–2.5 times higher than the average speed of the film. As a result of the braking effect of the heat exchange surface, large waves are deformed and further destroyed, changing the nature of the wave movement to the movement of a wedge-shaped thickening in the form of a shaft, the front part of which undergoes rotational movement, forming a central vortex. Under these conditions, during the passage of the shaft of a large wave, the superheated liquid of the wall part of the film is captured by a large vortex and transferred to the wave crest, where it gives heat to the vapor flow in the mode of self-evaporation. After the passage of a large wave, the wall part of the film becomes cooled due to the replacement of superheated liquid with liquid from the outer part of the film. Before the arrival of the next large wave, the liquid warms up, changing the temperature gradients on the wall and the interfacial surface, due to which the heat flux densities on the wall and the interfacial surface are different until the steady state of heat exchange occurs. The rate of heating of the film depends on the degree of turbulence by the vortex swept under a large wave. Recently, a number of algebraic ratios have been proposed for the steady turbulent motion of a film, which take into account the specifics of the development of turbulence in films (Alhusseini et al., 1998, 2000; Asbik et al., 2005; Peterson et al., 1997). A number of other turbulence models have been proposed previously, an overview of which is given in (Mascarenhas et al., 2013). A convenient model for finding an analytical expression for temperature and velocity profiles in the film is one that reflects in one dependence the presence of a wall laminar layer with a gradual transition to a turbulent core, and takes into account the rapid drop in turbulence intensity within the interfacial surface. The specified model was proposed in (Petrenko et al., 2020, 2023) and was used to analyze heat transfer in a film with a developed system of low-frequency large waves.

The aim of the present research was to theoretically and experimentally determine the influence of periodic disturbances of the liquid film surface on the intensity of heat transfer during vaporization.

Materials and methods

The object of research is heat transfer processes during forced disruption of the ordered hydrodynamic structure of a flowing liquid film. The subject of research is thermal-hydrodynamic processes in a flowing film of liquid, which is cyclically disturbed by surface large low-frequency waves or mechanical turbulators.

Methods

The theoretical results were obtained on the basis of mathematical modeling of heat transfer processes in flowing liquid films with disturbance of its surface by large waves generated in long heat exchange pipes of film evaporators. The solution of the equation of convective thermal conductivity was carried out by an approximate method with boundary conditions corresponding to the condition of vapor formation on the interfacial surface of the film.

Physical modeling was performed on an experimental stand, on which heat exchange processes in the film were reproduced in a 9-m-long pipe with a diameter of 30 mm. The model liquid is water saturated to the boiling point with a mass liquid flux of 0.16–0.72 kg/m· sec. Heating was carried out with dry saturated steam. The measurement of the heat flow was carried out by the volumetric method based on the amount of condensate formed on the sections of the heat exchange pipe with a height of 400 mm each.

Results and discussion

Heat exchange model

According to the proposed heat transfer model, it is postulated that after the passage of a large wave, the temperature profile "sags" due to the replacement of a part of the superheated liquid with one cooled from the interphase surface, and the depth of "sags" will be determined by the power of the central vortex. The limit curve to which the temperature profile "sags" depends on the degree of turbulence of the film at the moment of passage of the central vortex, and there is either no or a small temperature gradient on the interphase surface. The process of heat transfer per cycle is divided into two stages: first, there is a temperature profile, which is established after rapid cooling of the film as a result of removal of superheated liquid from the wall layer and its replacement with liquid from the area of the interfacial surface; then the process of restoring the temperature profile due to convective thermal conductivity to the previous level during the time interval between the passage of two large waves is considered. The temperature profile for both the first and second stages is found from the convective heat conduction equation, but under different boundary and initial conditions

$$u\frac{\partial t}{\partial x}=\frac{\partial}{\partial y}\left(a+a_t\right)\frac{\partial t}{\partial y}\ , \qquad (1)$$

where t – temperature; y , x – transverse and longitudinal coordinates; a – temperature conductivity; a_t – turbulent temperature conductivity; u – velocity of the liquid in the film.

Given that $u=u_i\left(\frac{y}{\delta}\right)^{\frac{1}{5}}=\frac{6}{5}\overline{u}\left(\frac{y}{\delta}\right)^{\frac{1}{5}}=\frac{6}{5}\overline{u}\,\eta^{\frac{1}{5}}$, in the dimensionless form (1) we write as

$$\frac{6\delta}{5}\bar{u}(\eta)^{1/5}\frac{\partial\theta(\xi,\eta)}{\partial\xi}=\frac{\partial}{\partial\eta}(a+a_t)\frac{\partial\theta(\xi,\eta)}{\partial\eta}, \tag{2}$$

where $\theta(\eta,\xi)=\frac{t(\eta,\xi)-t_o}{t_{cm}-t_o}$ – dimensionless temperature; t_{cm}, t_o – wall temperature and liquid saturation, respectively; $\eta=\frac{y}{\delta}$, $\xi=\frac{x}{\delta}$ – dimensionless transverse and longitudinal coordinates; δ – film thickness; $\bar{u}$ – the average velocity of the liquid in the film; u_i – velocity of liquid at the interphase boundary of the film.

Let's solve equation (2) using Targa's approximate method, replacing the left part of (2) with the average value

$$\int_0^1 \frac{6\delta}{5}\bar{u}(\eta)^{1/5}\frac{\partial\theta(\xi,\eta)}{\partial\xi}\partial\eta=\delta\,\bar{u}\,\frac{\partial\theta_{av}(\xi)}{\partial\xi},$$

Then from (2) we get

$$\frac{Pe}{4}\frac{\partial\theta_{av}(\xi)}{\partial\xi}=\frac{\partial}{\partial\eta}\left(1+\frac{a_t}{a}\right)\frac{\partial\theta(\xi,\eta)}{\partial\eta}, \tag{3}$$

where $Pe=\frac{4J_v}{a}$ the Peclet number; $J_v=\frac{G}{\rho\pi d}$ – volumetric liquid flux; G – mass flow rate; ρ – density; d – pipe diameter.

Since $\frac{a_t}{a}=\frac{\nu_t}{\nu}\frac{\Pr}{\Pr_t}$, expression (3) takes the form

$$\frac{Pe}{4}\frac{\partial\theta_{av}(\xi)}{\partial\xi}=\frac{\partial}{\partial\eta}\left(1+\frac{\nu_t}{\nu}\frac{\Pr}{\Pr_t}\right)\frac{\partial\theta(\xi,\eta)}{\partial\eta}. \tag{4}$$

where ν_t,ν – turbulent and molecular cinematic viscosity coefficient, respectively; $\Pr$, $\Pr_t$ – Prandtl number and turbulent Prandtl number, respectively.

Temperature profile in the film at the first stage of the wave cycle

On the interfacial surface ($\eta=1$) after the passage of a large wave, the temperature gradient is either too small or equal to zero, $\frac{\partial\theta}{\partial\eta}=0$, and the temperature corresponds to the saturation temperature. Under this boundary condition, the solution of (4) is a curve that determines the degree of "sags" of the temperature profile for a certain value of turbulent viscosity. In the wall region ($\eta=0$), the liquid temperature is equal to the wall temperature, and the initial conditions are formulated as $\xi=0$, $\theta=0$.

The first integration of (4) from the condition that at $\eta=1$, $\frac{\partial\theta}{\partial\eta}=0$, gives

$$\frac{Pe}{4}\frac{\partial\theta_{av}(\xi)}{\partial\xi}(\eta-1)=\left(1+\frac{\nu_t}{\nu}\frac{\Pr}{\Pr_t}\right)\frac{\partial\theta(\xi,\eta)}{\partial\eta}. \tag{5}$$

The function of turbulence in the film, which most adequately reflects the dependence of the turbulent viscosity distribution in the cross section of the film, will be given as (Petrenko et al., 2020).

$$\frac{\nu_t}{\nu} = \varepsilon_m \eta^2 \left(1-\eta^2\right) , \tag{6}$$

where ε_m is the maximum value of the turbulence function in the film at the top of the deformed parabola (correlation parameter).

Taking $\mathrm{Pr}_t = 1$, the general integral (5) taking into account (6) will be written as

$$\theta(\eta,\xi) = \frac{Pe}{4}\frac{\partial\theta_{av}(\xi)}{\partial\xi}\left[\int\frac{\eta\, d\eta}{\left(1+\varepsilon_m\left(\eta^2-\eta^4\right)\mathrm{Pr}\right)} - \int\frac{d\eta}{\left(1+\varepsilon_m\left(\eta^2-\eta^4\right)\mathrm{Pr}\right)}\right]. \tag{7}$$

Integrating (7) under the boundary conditions $\eta = 0$, $\theta = 1$, gives

$$\theta(\eta,\xi) = 1+\frac{Pe}{4}\frac{\partial\theta_{av}(\xi)}{\partial\xi}\left[\begin{array}{l}\frac{1}{H}\operatorname{atanh}\left(\frac{P\left(2\eta^2-1\right)}{H}\right)+\frac{\sqrt{2}H}{(4+P)A}\operatorname{atanh}\left(\frac{\sqrt{2}P}{A}\eta\right)- \\ -\frac{\sqrt{2}H}{(4+P)B}\operatorname{atanh}\left(\frac{\sqrt{2}P}{B}\eta\right)+\frac{1}{H}\operatorname{atanh}\left(\frac{P}{H}\right)\end{array}\right], \tag{8}$$

where $P = \varepsilon_m \mathrm{Pr}$; $H = \sqrt{4P+P^2}$; $A = \sqrt{P^2-PH}$; $B = \sqrt{P^2+PH}$.

The unknown derivative $\frac{\partial\theta_{av}(\xi)}{\partial\xi}$ is found from the relation for the average integral temperature of the film obtained from expression (8) under the initial condition $\xi = 0$, $\theta = 0$. Taking into account the fullness of the velocity profile for the turbulent film, with a certain approximation regarding the average temperature, we assume that

$$\theta_{cp}(\xi) = \int_0^1 \theta(\eta,\xi)\frac{u(\eta)}{\bar{u}}d\eta \approx \int_0^1 \theta(\eta,\xi)\,d\eta . \tag{9}$$

Integrating (8) gives:

$$\theta_{cp} = 1+\frac{\partial\theta_{cp}}{\partial\xi}\frac{Pe}{4}\left[\begin{array}{l}\frac{3\sqrt{2}P}{2H}\left(\frac{1}{A}\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)-\frac{1}{B}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)\right)- \\ -\frac{\sqrt{2}}{2}\left(\frac{1}{A}\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)+\frac{1}{B}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)\right)+ \\ +\frac{2}{H}\ln\left(\frac{B}{C}\right)+\frac{2}{H}\left(\frac{P}{H}\right)\end{array}\right], \tag{10}$$

where $C = \sqrt{PH-P^2}$.

Marking the expression in square brackets for S

$$S=\frac{Pe}{4}\left[\begin{array}{l}\frac{3\sqrt{2}P}{2H}\left(\frac{1}{A}\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)-\frac{1}{B}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)\right)-\frac{\sqrt{2}}{2}\left(\frac{1}{A}\left(\frac{P\sqrt{2}}{A}\right)+\frac{1}{B}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)\right)+\\+\frac{2}{H}\ln\left(\frac{B}{C}\right)+\frac{2}{H}\operatorname{atanh}\left(\frac{P}{H}\right)\end{array}\right] \tag{11}$$

we obtain the differential equation $\theta_{cp}=1+S\frac{d\theta_{cp}}{d\xi}$, the solution of which, under the initial condition $\xi=0$; $\theta=0$, is an expression

$$\theta_{cp}=\left[1-\exp\left(\frac{\xi}{S}\right)\right]. \tag{12}$$

It is substituted the derivative of (12) $\frac{d\theta_{cp}}{d\xi}=-\frac{1}{S}\exp\left(\frac{\xi}{S}\right)$ into (8)

$$\theta(\eta,\xi_m)=1-\frac{Pe}{4}\exp\left(\frac{\xi_m}{S}\right)\left[\begin{array}{l}\frac{1}{H}\operatorname{atanh}\left(\frac{P(2\eta^2-1)}{H}\right)+\frac{\sqrt{2}H}{(4+P)A}\operatorname{atanh}\left(\frac{\sqrt{2}\,P}{A}\eta\right)-\\-\frac{\sqrt{2}H}{(4+P)B}\operatorname{atanh}\left(\frac{\sqrt{2}\,P}{B}\eta\right)+\frac{1}{H}\operatorname{atanh}\left(\frac{P}{H}\right)\end{array}\right]. \tag{13}$$

The obtained temperature profile (13) is valid only for one curve for a specific value of the longitudinal coordinate $\xi=\xi_m$, under which the condition is fulfilled $\frac{\partial\theta}{\partial\eta}=0$. Formally, the depth of "sags" of the temperature profile (13) is determined by the parameter ε_m , which is proportional to the degree of turbulence of the film at the moment of passage of the central vortex, Figure 1.

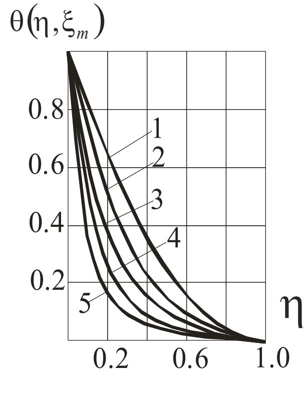

Figure 1. Limit temperature curve (13).
1 – ε_m = 0.01; 2 – 3; 3 – 10; 4 – 30; 5 – 80.

The coordinate ξ_m, at which $\frac{\partial\theta}{\partial\eta}=0$, is found from the condition $\eta=1$, $\theta(1,\xi_m)=0$ from equation (13)

$$\xi_m = S\ln\left(\frac{4S}{Pe\,K}\right), \tag{14}$$

where $K=\begin{bmatrix}\frac{2}{H}\text{atanh}\left(\frac{P}{H}\right)+\frac{\sqrt{2}H}{(4+P)A}\text{atanh}\left(\frac{\sqrt{2}\,P}{A}\right)- \\ -\frac{\sqrt{2}H}{(4+P)B}\text{atanh}\left(\frac{\sqrt{2}\,P}{B}\right)\end{bmatrix}$.

Temperature profiles in the film at the stage of equalization of the temperature field in the second stage of the wave cycle

In the region $\xi \geq \xi_m$, the temperature of the interfacial surface of the film due to evaporation remains constant along the longitudinal coordinate ξ, and the temperature profile develops from the limit curve (13) to a straight line (in the flat setting) at $\xi \to \infty$. Since the temperature of the interfacial surface in the region $\xi \geq \xi_m$ remains constant, we will determine the temperature field from equation (4) under boundary conditions

$$\eta=0,\theta=1;\quad \eta=1,\ \theta=0. \tag{15}$$

The initial condition is the limiting curve (13), with $\xi=\xi_m$.

Double integration (4) with the turbulent viscosity distribution function (6) and boundary conditions (15) gives

$$\begin{gathered}\theta(\eta,\xi)=\frac{Pe}{4}\frac{\partial\theta_{cp}}{\partial\xi}\left[\frac{1}{H}\text{atanh}\left(\frac{2P\eta^2-P}{H}\right)\right]+ \\ +\left[\frac{-H\sqrt{2}}{(4+P)A}\text{atanh}\left(\frac{P\sqrt{2}}{A}\eta\right)+\frac{H\sqrt{2}}{(4+P)B}\text{atanh}\left(\frac{P\sqrt{2}}{B}\eta\right)\right]\times \\ \times\frac{\frac{-Pe}{2H}\frac{\partial\theta_{cp}}{\partial\xi}\text{atanh}\left(\frac{P}{H}\right)-1}{\frac{H}{4+P}\left(\frac{\sqrt{2}}{B}\text{atanh}\left(\frac{P\sqrt{2}}{B}\right)-\frac{\sqrt{2}}{A}\text{atanh}\left(\frac{P\sqrt{2}}{A}\right)\right)}+\frac{Pe}{4H}\frac{\partial\theta_{cp}}{\partial\xi}\text{atanh}\left(\frac{P}{H}\right)+1.\end{gathered} \tag{16}$$

It is found the unknown derivative $\dfrac{\partial \theta_{av}(\xi)}{\partial \xi}$ from (16) using the initial condition (13). Let's substitute in (16) the average value of the temperature, which will be found by integration $\theta_{cp}(\xi) = \int_0^1 \theta(\eta,\xi)\dfrac{u(\eta)}{\bar{u}}d\eta \approx \int_0^1 \theta(\xi,\eta)d\eta$

$$\theta_{cp} = W\frac{\partial \theta_{cp}}{\partial \xi} + Z \text{ , (17)}$$

where

$$W = \frac{Pe\sqrt{2}}{N}\left\{\begin{aligned}&\left[\left(\frac{1}{8c}-\frac{P}{8cH}\right)\operatorname{atanh}\left(\frac{P\sqrt{2}}{c}\right)\left(\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)B-\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)A\right)\right]+\\&+\left[\frac{1}{8B}+\frac{P}{8BH}\right]\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)\left(\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)B-\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)A\right)+\\&+\frac{AB}{8PH}\operatorname{atanh}\left(\frac{P}{H}\right)\ln\left[\frac{\left(A^2-2P^2\right)B^2}{\left(B^2-2P^2\right)A^2}\right]\end{aligned}\right\}, \quad (18)$$

$$Z = \frac{\sqrt{2}AB}{4NP}\ln\left[\frac{\left(A^2-2P^2\right)B^2}{\left(B^2-2P^2\right)A^2}\right], \quad (19)$$

$$c = \sqrt{P^2 - PH}\text{ ; } N = A\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right) - B\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right).$$

The solution of (17) is

$$\theta_{cp} = Z + c_3\exp\left(\frac{\xi}{W}\right). \quad (20)$$

The constant of integration c_3 is found from the initial condition (13), from which we determine the average value $\theta_{cp.m} = \int_0^1 \theta(\eta,\xi_m)d\eta$ by integration when $\xi = \xi_m$.

$$\theta_{cp.m}=1+\left[\frac{Pe\exp\left(\frac{\xi_m}{S}\right)}{S(P+4)}\right]\left[\begin{array}{l}\frac{H\sqrt{2}}{4}\left(\frac{1}{B}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)-\frac{1}{A}\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)\right)+\sqrt{2}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)\times\\ \times\left[\frac{1}{2B}+\frac{P}{8B}+\frac{P}{2HB}+\frac{P^2}{8HB}\right]+\left(\frac{H}{8P}\right)\left[\ln\left(\frac{\left(B^2-2P^2\right)A^2}{\left(A^2-2P^2\right)B^2}\right)\right]+\\ +\sqrt{2}\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)\left[\frac{1}{2A}+\frac{P}{8A}-\frac{P}{2Ha}-\frac{P^2}{8HA}\right]-\\ \operatorname{atanh}\left(\frac{P}{H}\right)\left(\frac{2}{H}+\frac{P}{2H}\right)\end{array}\right]. \quad (21)$$

Then (20) will be rewritten as $\theta_{cp.m}=Z+c_3\exp\left(\frac{\xi_m}{W}\right)$, where is the constant of integration

$$c_3=\left(\theta_{cp.m}-Z\right)\exp\left(\frac{-\xi_m}{W}\right).$$

By substituting c_3 in (20), we get the average temperature in the film along the coordinate ξ

$$\theta_{cp}(\xi)=Z+\left(\theta_{cp.m}-Z\right)\exp\left(\frac{\xi-\xi_m}{W}\right). \quad (22)$$

Derivative of θ_{cp} by ξ

$$\frac{\partial\theta_{cp}}{\partial\xi}=\left(\frac{\theta_{cp.m}-Z}{W}\right)\exp\left(\frac{\xi-\xi_m}{W}\right). \quad (23)$$

Substitute the resulting derivative into (16) and obtain the final expression for the temperature field in the film in the region $\xi\geq\xi_m$

$$\begin{aligned}\theta(\eta,\xi)=&\frac{Pe}{4H}\left(\frac{\theta_{cp.m}-Z}{W}\right)\exp\left(\frac{\xi-\xi_m}{W}\right)\operatorname{atanh}\left(\frac{2P\eta^2-P}{H}\right)+\\ &+\left(\frac{H\sqrt{2}}{4+P}\left(\frac{1}{B}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\eta\right)-\frac{1}{A}\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\eta\right)\right)\right)\times\\ &\times\left[\frac{\frac{-Pe}{2H}\left(\frac{\theta_{cp.m}-Z}{W}\right)\exp\left(\frac{\xi-\xi_m}{W}\right)\operatorname{atanh}\left(\frac{P}{H}\right)-1}{\frac{H\sqrt{2}}{(4+P)}\left[\frac{1}{B}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)-\frac{1}{A}\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)\right]}\right]+\frac{Pe}{4H}\left[\left(\frac{\theta_{cp.m}-Z}{W}\right)\exp\left(\frac{\xi-\xi_m}{W}\right)\operatorname{atanh}\left(\frac{P}{H}\right)+1\right]\end{aligned}$$

(24)

The graphic interpretation of the development of the temperature field according to (24) at different distances from ξ_m and with different degrees of film turbulence ε_m is shown in Figure 2.

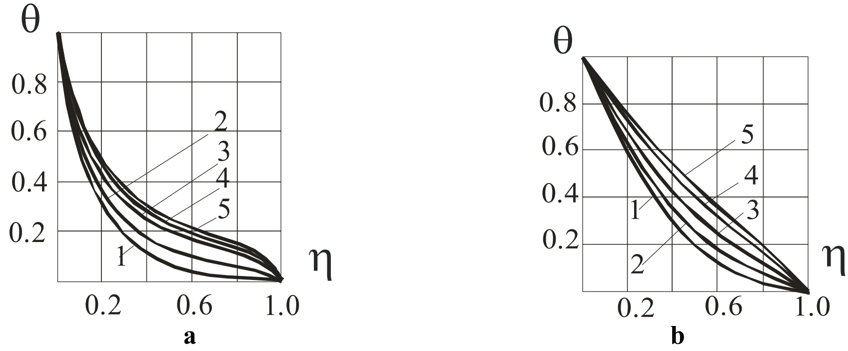

Figure 2. Leveling of the temperature profile after perturbation of the film by a large wave at $Pe = 7000$**;** $\Pr = 2$.

a. $\varepsilon_m = 20$; $\xi_m = 51.9$; 1 – $\xi = \xi_m$; 2 – $\xi = 70$; 3 – 110; 4 – 160; 5 – 800.

b. $\varepsilon_m = 2$; $\xi_m = 187.1$; 1 – $\xi = \xi_m$; 2 – $\xi = 220$; 3 – 280; 4 – 400; 5 – 800.

Heat flux on the heat exchange surface and heat transfer coefficient

The temperature gradient on the wall $\frac{d\theta(\eta,\xi)}{d\eta}\Big|_{\eta=0}$ changes from the maximum at the time of passage of the vortex of a large wave at $\xi = \xi_m$ to the minimum asymptotic at $\xi = \infty$. The derivative of (24) on the wall is equal to

$$\frac{d\theta(\eta,\xi)}{d\eta}\Big|_{\eta=0} = \frac{2HP}{4+P}\left(\frac{1}{B^2}-\frac{1}{A_2}\right)\frac{\frac{-Pe}{2H}\left(\frac{\theta_{cp.m}-Z}{W}\right)\exp\left(\frac{\xi-\xi_m}{W}\right)\operatorname{atanh}\left(\frac{P}{H}\right)-1}{\frac{H\sqrt{2}}{4+P}\left[\frac{1}{B}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)-\frac{1}{A}\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)\right]}. \tag{25}$$

From the expression $q(\xi)_{\eta=0} = -\lambda\frac{t_w-t_{sat}}{\delta}\frac{d\theta}{d\eta}_{\eta=0}$, we find the heat flux density on the wall

$$q(\xi)\big|_{\eta=0} = -\lambda\frac{t_w-t_{sat}}{\delta}\frac{2HP}{4+P}\left(\frac{1}{B^2}-\frac{1}{A_2}\right)\frac{\frac{-Pe}{2H}\left(\frac{\theta_{cp.m}-Z}{W}\right)\exp\left(\frac{\xi-\xi_m}{W}\right)\operatorname{atanh}\left(\frac{P}{H}\right)-1}{\frac{H\sqrt{2}}{4+P}\left[\frac{1}{B}\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)-\frac{1}{A}\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)\right]}, \tag{26}$$

where λ – heat conduction of liquid.

The average heat flux on the wall in the interval between the passage of two large waves

$$q_{cp}=\frac{1}{\xi_v-\xi_m}\int_{\xi_m}^{\xi_v}q(\xi)_{\eta=0}\,d\xi=-\frac{\lambda}{\delta}\frac{t_w-t_{sat}}{\xi_v-\xi_m}\left[\frac{P}{ABH}\frac{\sqrt{2}\left(A^2-B^2\right)}{B\cdot\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)-A\cdot\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)}\right]\times$$
$$\times\left[\frac{Pe}{2}\operatorname{atanh}\left(\frac{P}{H}\right)\left(\theta_{cp.m}-Z\right)\left(\exp\left(\frac{\xi_v-\xi_m}{W}\right)-1\right)+H\left(\xi_v-\xi_m\right)\right]. \quad (27)$$

Heat transfer coefficient

$$\alpha=\frac{q_{cp}}{t_w-t_{sat}}=-\frac{\lambda}{\delta}\frac{1}{\xi_v-\xi_m}\left[\frac{P}{ABH}\frac{\sqrt{2}\left(A^2-B^2\right)}{B\cdot\operatorname{atanh}\left(\frac{P\sqrt{2}}{A}\right)-A\cdot\operatorname{atanh}\left(\frac{P\sqrt{2}}{B}\right)}\right]\times$$
$$\times\left[\frac{Pe}{2}\operatorname{atanh}\left(\frac{P}{H}\right)\left(\theta_{cp.m}-Z\right)\left(\exp\left(\frac{\xi_v-\xi_m}{W}\right)-1\right)+H\left(\xi_v-\xi_m\right)\right]. \quad (28)$$

The thickness of the film, which is included in expression (28), is found from the equation of motion of a freely flowing film using the turbulence function (6).

$$\frac{g\delta^2}{\nu}(1-\eta)=\left[1+\varepsilon_m\left(\eta^2-\eta^4\right)\right]\frac{du}{d\eta}. \quad (29)$$

where g – acceleration of gravity.

The solution of (29) under the boundary condition $y=0$, $u=0$ is the velocity profile

$$u=\left(\frac{g\delta^2}{\nu}\right)\frac{\sqrt{2}h}{\left(4+\varepsilon_m\right)}\left[\frac{1}{b}\operatorname{atanh}\left(\frac{\sqrt{2}\varepsilon_m}{b}\eta\right)-\frac{1}{r}\operatorname{atanh}\left(\frac{\sqrt{2}\varepsilon_m}{r}\eta\right)\right]-$$
$$-\frac{g\delta^2}{\nu h}\left[\operatorname{atanh}\left(\frac{\varepsilon_m\left(2\eta^2-1\right)}{h}\right)+\operatorname{atanh}\left(\frac{\varepsilon_m}{h}\right)\right].$$

Average speed in the film

$$\overline{u}=\left(\frac{g\delta^2}{\nu}\right)\frac{\sqrt{2}h}{\left(4+\varepsilon_m\right)}\left[n-\frac{\sqrt{2}}{4\varepsilon_m}\ln\left(\frac{r^2-2\varepsilon_m^2}{b^2-2\varepsilon_m^2}\right)-\frac{\sqrt{2}}{2\varepsilon_m}\ln\left(\frac{b}{r}\right)\right]-$$
$$-\frac{g\delta^2}{\nu h}\left[2\operatorname{atanh}\left(\frac{\varepsilon_m}{h}\right)-\frac{\sqrt{2}h}{2}\left(\frac{1}{b}\operatorname{atanh}\left(\frac{\sqrt{2}\varepsilon_m}{b}\right)+\frac{1}{r}\operatorname{atanh}\left(\frac{\sqrt{2}\varepsilon_m}{r}\right)\right)-\frac{\sqrt{2}\varepsilon_m}{2}n\right].$$

Where do we get the thickness of the film, given that the mass liquid flux $J = \rho \bar{u} \delta$,

$$\delta = \sqrt[3]{\frac{J\nu h}{\rho g(Dh - R)}} \quad , \tag{30}$$

where $R = \left[2\,\mathrm{atanh}\left(\frac{\varepsilon_m}{h}\right) - \frac{\sqrt{2}h}{2}\left(\frac{1}{b}\mathrm{atanh}\left(\frac{\sqrt{2}\varepsilon_m}{b}\right) + \frac{1}{r}\mathrm{atanh}\left(\frac{\sqrt{2}\varepsilon_m}{r}\right)\right) - \frac{\sqrt{2}\varepsilon_m}{2}n \right]$,

$$D = \frac{\sqrt{2}\,h}{(4+\varepsilon_m)}\left[n - \frac{\sqrt{2}}{4\varepsilon_m}\ln\left(\frac{r^2 - 2\varepsilon_m^2}{b^2 - 2\varepsilon_m^2}\right) - \frac{\sqrt{2}}{2\varepsilon_m}\ln\left(\frac{b}{r}\right) \right],$$

$$h = \sqrt{4\varepsilon_m + \varepsilon_m^2}\ , \quad r = \sqrt{\varepsilon_m^2 - \varepsilon_m h}\ , \quad b = \sqrt{\varepsilon_m^2 + \varepsilon_m h}\ ;$$

$$n = \left[\frac{1}{b}\mathrm{atanh}\left(\frac{\sqrt{2}\,\varepsilon_m}{b}\right) - \frac{1}{r}\mathrm{atanh}\left(\frac{\sqrt{2}\,\varepsilon_m}{r}\right) \right].$$

The intensity of heat transfer, as can be seen from (28), is inversely proportional to the distance $\xi_v - \xi_m$, which is interpreted as the dimensionless distance between the crests of large waves, in the case of a flow with a developed wave structure, or the distance between mechanical turbulators, in the case of their application. The results of calculating the heat transfer coefficient according to the ratio (28) depending on the distance between the heat exchange intensifiers L at the value of the parameter ε_m = 10.4 are shown in Figure 3.

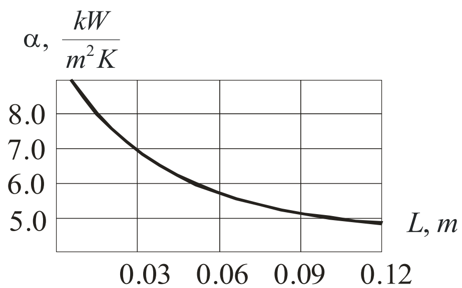

Figure 3. Dependence $\alpha = f(L)$ **according to relation** (28) **at** ε_m = 10.4, J =0.3 $\frac{kg}{m \cdot \sec}$

The degree of film turbulence is determined by the power of the vortex of a large wave, in the case of a flow with a developed wave structure, or the height of a mechanical turbulator, in the case of their application. The distance between the crests of large waves for pipes with a diameter of 25–35 mm is about 0.12 m. The results of calculating the heat transfer coefficient according to the ratio (28) depending on the degree of film turbulence are shown in Figure 4.

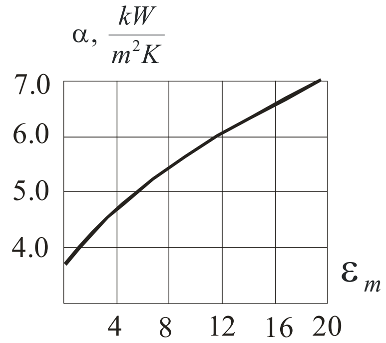

Figure 4. Dependence according to relation (28) at $L = 0.12\ m,\ J = 0.3\ \frac{kg}{m \cdot \sec}$

Thus, the obtained ratios adequately reflect the dependence of the heat transfer coefficient to the flowing liquid films during vaporization with heat exchange intensifiers on the turbulence parameters. The data of experimental studies of heat transfer to flowing films of water in the mode of evaporation from a free surface under atmospheric pressure in a pipe 9 m long with a diameter of 30 mm, where a developed structure of large waves with a length of 0.12 m is formed at a distance of 2 m, correspond to the theoretical results obtained, calculated by relation (28). A comparison of experimental and calculated results on the intensity of heat exchange established that their correspondence takes place under the condition of dependence ε_m on the mass liquid flux J in the form:

$$\varepsilon_m = 0.634 - 4.348J + 158.175J^2 + 394.98J^3 - 2844J^4 + 4823J^5 - 2645J^6 \qquad (31).$$

Note that the obtained dependence (31) cannot be considered as a turbulence parameter, since it was obtained as a correlation parameter of heat transfer data, and not because of direct measurements of turbulent pulsations. Graphical interpretation of dependence (31) is shown in Figure 5.

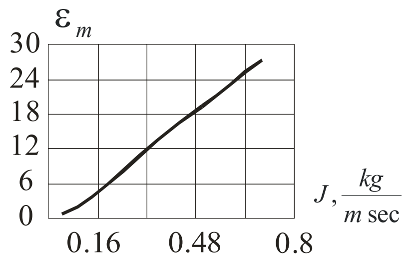

Figure 5. Dependence $\varepsilon_m = f(J)$ **according to ratio (31)**

As can be seen from the graph in fig. 5, in the turbulent region with a mass liquid flux greater than 0.16 kg/m sec, which corresponds to a Reynolds number of 2200, the dependence $\varepsilon_m = f(J)$ is almost linear. A comparison of the experimental results with those calculated using the ratios (28, 31) is shown in Figure 6.

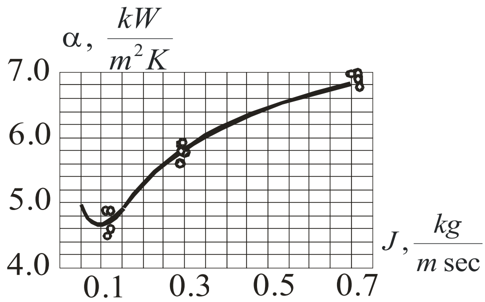

Figure 6. Dependence $\alpha = f(J)$ for water at atmospheric pressure in the region of the developed structure of large waves in the mode of evaporation from the interfacial surface in a pipe with a diameter of 30 mm. Line – calculation according to (28, 31).

Conclusions

1. An approximate analytical solution of the convective heat conduction equation for a turbulent film flowing down a vertical heat exchange surface was performed, and an analytical expression for the temperature field in the film, which develops in the transverse and longitudinal directions, was obtained.
2. The developed mathematical model of heat transfer in a film of liquid flowing down a vertical surface, in which the cyclic mixing of the film by low-frequency waves of large amplitude is considered, allows modeling the processes of heat transfer in a film of liquid during vaporization under the condition of periodic forced disruption of its structure.
3. The ratios obtained because of the approximate solution of the equations of convective heat conduction reflect the regularities of heat transfer in flowing films in the regime of both its periodic mixing by large waves and the system of passive ring intensifiers located with different densities.

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